# Experiment 8
# Curve Fits (Trendlines)

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Name:



Date:

## Key Objectives

1. Apply a curve fit (trendline) to data in a graph.
2. Use of curve-fits (trendlines) to solve for missing variable.
3. Use of linear, exponential and power equations.
4. Drawing good graphs.

## Discussion

The goal of this lab is to extend the lessons learned making graphs and learn several new skills including how to read a graph, extrapolation, curve-fitting (trend-lines), and the use of equations of lines.

A set of data taken in lab is nice, but without a good way of interpreting that data it is less than useful. One of the best ways to interpret data is using graphical methods and mathematical equations. Qualitatively graphs allow one to quickly and visualize the data, noting whether the relationship between two variable is directly proportional or inversely proportion, or even an more complicated relationship.

A mathematical equation provides a quantitatively way to look at data, and complements a graph by allowing interpolation and extrapolation of the data between the points.

In several laboratory investigations you do this year, one of the primary purposes will be to find the mathematical relationship between the measured variables. This is useful in many ways including understanding the phenomenon and allows prediction of values not measured.

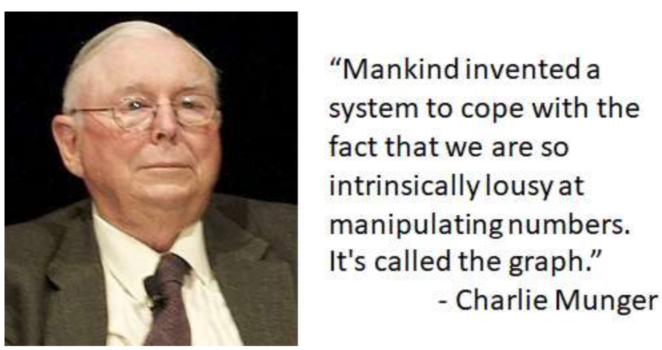


Figure 8.1: A picture is worth a thousand words, then a graph and an equation is worth 10,000. credit: `https://en.wikipedia.org/wiki/Charlie_Munger`

### Equations of Lines

Mathematically we describe a line using several different methods (Table 1 below). We will explore each type of line in more detail below.

| Trend/Regression Type | General Equation | Example |
|---|---|---|
| Linear | $y = mx + b$ | Figure 8.2 |
| Exponential | $y = ae^x$ | Figure 8.3 |
| Power | $y = ax^b$ | Figure 8.4 |
| Polynomial. | Never ever use this! | |

Table 8.1: Equations for Curve-fitting

## Linear Equations

Most students should be familiar with the form of a linear equation, but a brief review is in order. The graph of a generic linear equation in Figure8.2 show the relationship between the x and y data.

The change in the x and y values is called the slope (m) and mathematically is defined below in equation (1). The slope of the line can be determined by using the slope equation and two points, but more commonly we will determine the slope by using computer and determining a trend-line.

$$slope(m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run} = \frac{\Delta y}{\Delta x} \tag{4}$$

If the slope of the line increases from left to right, it indicates a direct relationship between x and y ($x \propto y$), while a line that decreases from left to right indicates inverse relationship ($x \propto \frac{1}{y}$).

The value at which x = 0, y = #, is called the y-intercept (b) in the equation. This value is sometimes relevant and useful, and sometimes simply part of the equation.

Linear Equation
($y = 2x + 10$)

| X-Values (unit) | Y-Values (unit) |
|---|---|
| 0.0 | 10.0 |
| 2.0 | 14.5 |
| 4.0 | 19.0 |
| 6.0 | 23.5 |
| 8.0 | 28.0 |
| 10. | 32.5 |

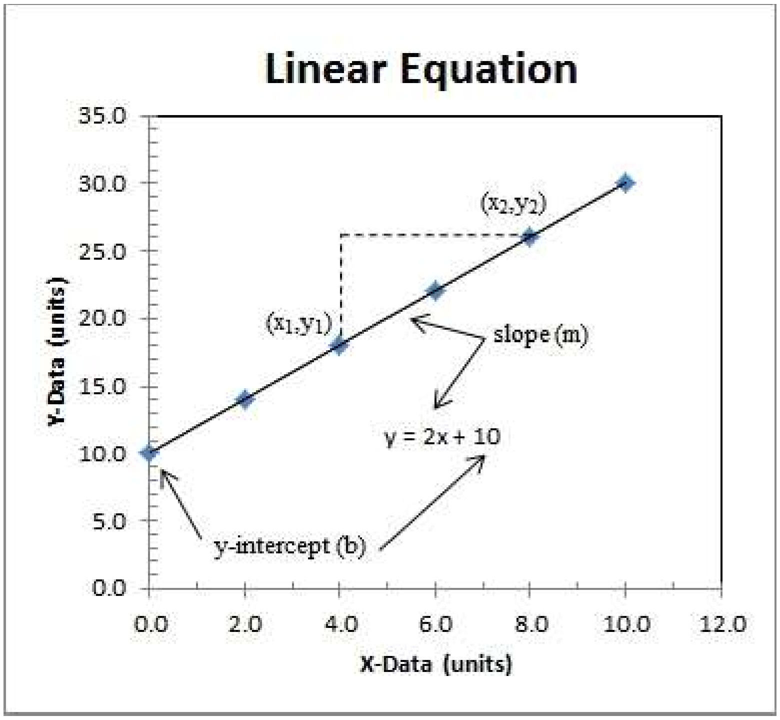


Figure 8.2: Data and Example Plot for a Linear Equation. credit: author

## Exponential Equations

In general, an exponential function is one of the form $a \cdot b^x$, where the base is "$b$" and the exponent is "$x$". However, nowadays the term exponential function is almost exclusively used as a short-cut for the natural exponential function $e^x$. For clarity we will call functions with a base of $e$ an exponential function, and a base of $x$ a power function.

The standard equation $y = ax^b$ can be used to describe exponential growth $(b > 0)$ and decay $(b < 0)$ where $a$ is the initial amount. In the graphs below one can see the effect of changing the $a$ and $b$ variables. The $a$ variable shifts the graph up or down in value, while changing the $b$ variable causes the graph to increase quickly in value. A negative sign for the exponent $(-b)$ results in an exponential decay.

| **X-Values** | **Equation 1** $y = e^{0.25x}$ | **Equation 2** $y = 2 * e^{0.25x}$ | **Equation 3** $y = e^{0.5x}$ | **Equation 4** $y = 30e^{-0.25x}$ |
|---|---|---|---|---|
| 1.0 | 1.284 | 2.568 | 1.648 | 23.36 |
| 2.0 | 1.649 | 3.297 | 2.718 | 18.20 |
| 4.0 | 2.718 | 5.437 | 7.389 | 11.04 |
| 6.0 | 4.481 | 8.963 | 20.09 | 6.694 |
| 8.0 | 7.389 | 14.78 | 54.60 | 4.060 |
| 10.0 | 12.18 | 24.36 | 148.4 | 2.162 |

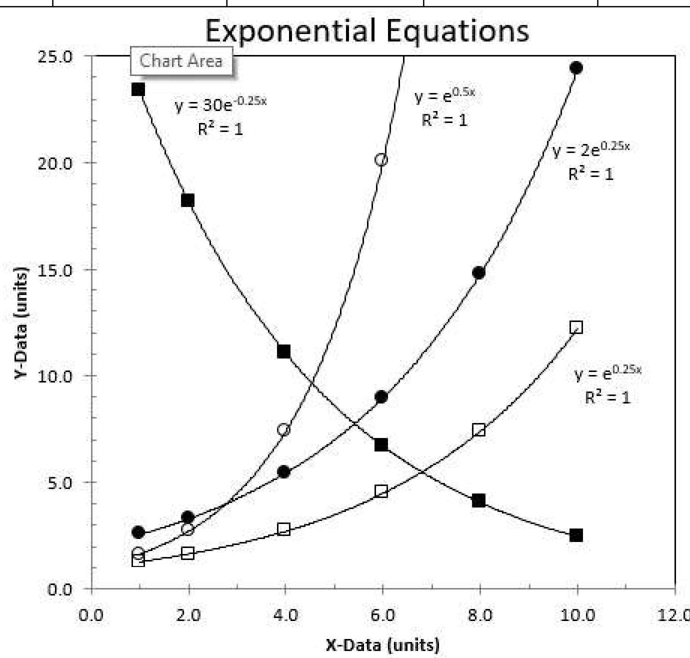


Figure 8.3: Data and Example Plots for Four Exponential Equations. credit: author.

Exponential equations can be used in real life to describe bacterial growth (or decay), and in chemistry to model the decay of some radioactive elements.

## Power Equations

Power equations are a form of exponential equation with the form $y = ax^b$. Like exponential equations they can be used to model both increasing and decreasing functions.

Power Equation ($y = x^2$)

| **X-Values** (unit) | **Y-Values** (unit) |
|---|---|
| 1.0 | 1.0 |
| 2.0 | 4.5 |
| 4.0 | 16.0 |
| 6.0 | 36.0 |
| 8.0 | 64.0 |
| 10. | 100.0 |

Power Equation ($y = x^3$)

| **X-Values** (unit) | **Y-Values** (unit) |
|---|---|
| 1.0 | 1 |
| 1.5 | 3.375 |
| 2.0 | 8 |
| 3.0 | 27 |
| 4.0 | 64 |
| 5.0 | 125 |

Power Equation ($y = 100x^{-2}$)

| **X-Values** (unit) | **Y-Values** (unit) |
|---|---|
| 1.0 | 100. |
| 2.0 | 25.0 |
| 3.0 | 1.11 |
| 4.0 | 6.25 |
| 5.0 | 4.00 |
| 6.5 | 2.78 |
| 7.0 | 2.04 |
| 8.0 | 1.56 |
| 9.0 | 1.56 |
| 10.0 | 1.00 |

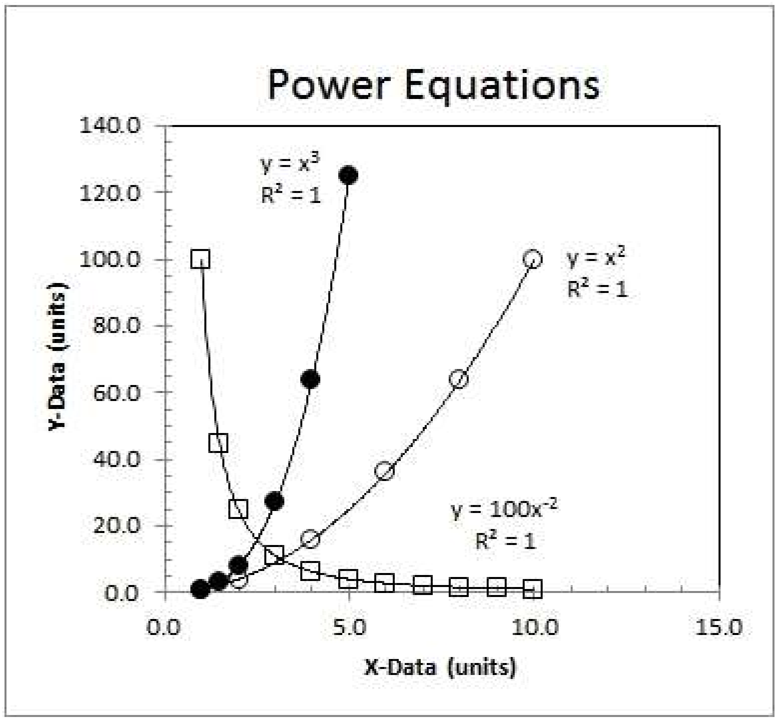


Figure 8.4: Data and Example Plots for Three Power Equations

## Using a Trend-Line

A Trend-Line is a mathematical relationship between two variables. Having a Trend-Line allows you to take information about the variable on the x-axis and convert it to the value on the y-axis or vice versa.

### Linear Equations

For a linear equation ($y = mx + b$) the relationship uses basic Algebra to convert between the variable. For example your lawyer might charge you an initial fee of $200.0 and $75.0 per hour after that. The equation of a line describing the billing would be $y = 75.0x + 200$, where the cost is plotted on the $y$-axis and the time consulting (in hours) is plotted on the $x$-axis. If the lawyer works on your case for 10.0 hours your total cost is given by:

$$y = 75.00x + 200.00$$
$$y = \frac{75.00 \text{ dollars}}{\text{hour}} \times \frac{10 \text{ hours}}{} + 200.00 \text{ dollars}$$
$$y = 950 \text{ dollars}$$

A slightly more cynical case might be if your budget for a lawyer is only 2000 dollars, how many hours can you hire a lawyer for.

$$y = 75.00x + 200.00$$
$$2000 = \frac{75.00 \text{ dollars}}{\text{hour}} \times \frac{x \text{ hours}}{} + 200 \text{ dollars}$$
$$x = 24 \text{ hours}$$

### Power Equations

Solving for variable in a power equation ($y = x^b$) requires the use of two keys on your calculator the $x^y$ key and its inverse the $x^{\frac{1}{y}}$ key. The melting point of the alkali metals can be described by the following equation, $y = 398x^{-0.639}$ where $y$ = atomic number and $x$ = melting point in ℃. If the atomic mass of Lithium is 3, what is the melting point in ℃? Use the $x^y$ key to enter the formula.

$$y = 398x^{-0.639}$$
$$y = 398 \cdot 3^{-0.639}$$
$$y = 197.3\ ^{\circ}\text{C}$$

If instead of the atomic mass we are given melting point of a compound we can calculate its atomic number and see how good the formula is. Cesium has a melting point of 28.5 ℃, how does the atomic number calculated from the equation compare to the actual value of 55? You will need to use the $x^{\frac{1}{y}}$ to solve the problem.

$$
\begin{aligned}
y &= 398x^{-0.639} \\
28.5 &= 398 \cdot x^{-0.639} \\
0.0716 &= x^{-0.639} \\
x &= 62 \text{ amu}
\end{aligned}
$$

It turns out that the equation is not very good!

**Exponential Equations**

Solving for variable in exponential equations ($y = e^x$) is straight forward for when solving for a missing $y$-value, but solving for $x$ requires a bit more math than is typically required in class. The opposite of the exponential function is the natural log function ($ln$). As an example, the rate at which a bacterial colony is growing is given by the equation $y = 2000e^x$ where 2000 = initial number of bacteria, $y$ = number of bacteria at time (x), and $x$ is the time in hours. How long (in hours) will it take for the colony to reach 100,000 bacteria?

$$
\begin{aligned}
y &= 2000e^x \\
100,000 &= 2000e^x \\
50 &= e^x \\
ln(50) &= x \\
x &= 3.91 \text{ hours}
\end{aligned}
$$

## Making Good Tables

Consult Lab 2 for a thorough review of making a good graph. Below is a short check-list to use when making a graph. An example of two data tables is shown below.

1. Tables may be in columns (as shown in previous figures) or rows (shown below), chose whichever makes the presentation of the data look the nicest.
2. Clearly label each column (or row) of data. Make sure to include units.
3. Include a title for the data describing it.

Data from Experiment 1 - Absorbance Spectrum of an Unknown Solution

| Wavelength (nm) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Absorbance | 1.6 | 2.7 | 4.4 | 6.4 | 8.9 | 13.1 | 19.3 | 28.2 | 38.2 | 48.7 |

Figure 8.5: Example of a Horizontal Table

## Making Good Graphs

Consult Lab 2 for a thorough review of making a good graph. Below is a short check-list (in no particular order) to use when making a graph.

1. Use the x-axis for the independent variable (that which is experimentally varied; also known as the manipulated variable) and the y-axis for the dependent variable (that which is a function of the independent variable; also known as the responding variable).
2. Label both the $x$ and $y$-axis. Be sure to include units.
3. Include major and minor ticks on each axis, chose reasonable spaced units for them.
4. Show the proper number of Significant Figures for the data on each axis.
5. Include a title.
6. Symbols should be properly sized.
7. Make sure the graph is black and white.
8. If more than one set of data is plotted include a Legend.
9. Do not put grid lines on your graph.
10. For the greatest accuracy, select scales so that the graph fills the page.
11. The data points should fill up the entire graph; they are not all bunched together in one corner.
12. Some examples of good graphs can be found in Lab 2, Lab 10, and this lab.

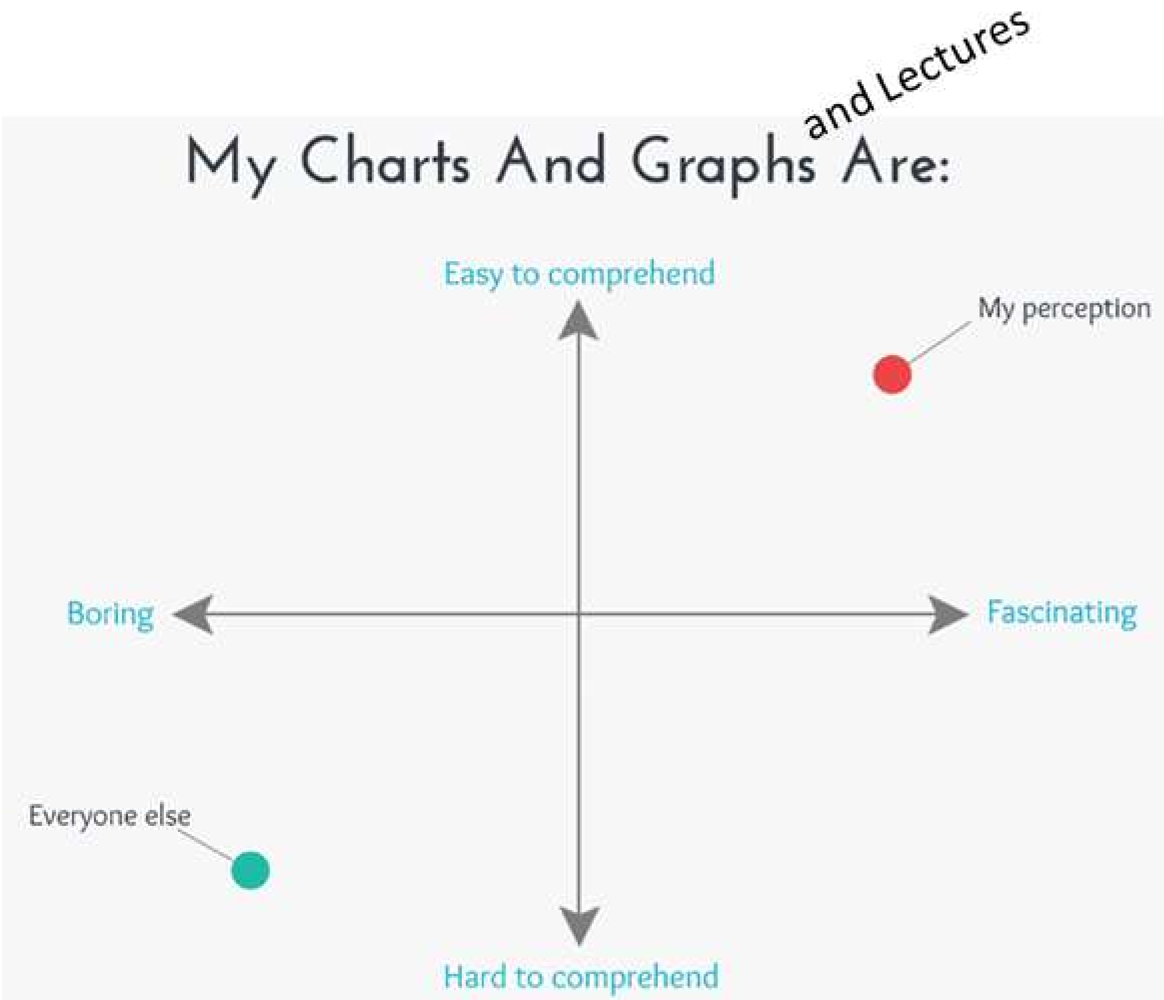


Figure 8.6: Just filling up some space but it is kinda sad but true. credit: unknown

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Name: ______________________ Class: ________________ Date: ________________

## Results

**For each of the following questions, graph the data and attach the graph to the back of the lab.**

### Problem 1: Determining the Density of an Unknown

The data in the table below are a result of experiments by a scientist on an unknown substance. The volume of substance was varied and the mass of each sample was measured. The slope of the graph will give the density of the unknown substance ($D = M/V$). Create a plot with volume on the x-axis, and mass on the y-axis, showing the mass of an unknown substance as a function of volume. Fit a linear equation to the line and determine the identity of the unknown substance. Be sure to include **UNITS** where appropriate.

| Volume (mL) | Mass (g) |
|---|---|
| 20.0 | 10.0 |
| 50.0 | 25.0 |
| 96.0 | 50.0 |
| 116. | 60.0 |
| 156 | 80.0 |
| 215 | 110.0 |

1. What is the equation of the line drawn:

2. What is the most probable identity of the unknown substance? (Explain how you made your choice).

### Problem 2: Temperature

The following data describes the relationship between the Celsius and Fahrenheit temperature scales. Graph the data, apply a linear curve fit and answer the questions below.

| Temp. (°C) | Temp. (°F) |
|---|---|
| 0 | 32 |
| 20 | 68 |
| 37 | 99 |
| 50 | 122 |
| 100 | 212 |

1. What is the equation of the line drawn:

2. What is the significance of equation of the line drawn?

## Selecting the Correct Trend-Line

Using the individual data set assigned to you (by your instructor), graph the data, and select the best trend-line to describe it. You **DO NOT** have to turn in your graphs, only fill in the data table on the following page. Be sure to include which data set you were assigned. You may need to try more than one Trend-line to see which provides the best fit.

Data Set 1:

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 7 | 9 | 11 | 13 | 15 | 17 | 19 | 21 | 23 | 25 | 27 | 29 |
| 2 | 9 | 11.25 | 13.5 | 15.75 | 18 | 20.25 | 22.5 | 24.75 | 27 | 29.25 | 31.5 | 33.75 |
| 3 | 11 | 13.5 | 16 | 18.5 | 21 | 23.5 | 26 | 28.5 | 31 | 33.5 | 36 | 38.5 |
| 4 | 13 | 15.75 | 18.5 | 21.25 | 24 | 26.75 | 29.5 | 32.25 | 35 | 37.75 | 40.5 | 43.25 |
| 5 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 | 39 | 42 | 45 | 48 |
| 6 | 17 | 20.25 | 23.5 | 26.75 | 30 | 33.25 | 36.5 | 39.75 | 43 | 46.25 | 49.5 | 52.75 |
| 7 | 19 | 22.5 | 26 | 29.5 | 33 | 36.5 | 40 | 43.5 | 47 | 50.5 | 54 | 57.5 |
| 8 | 21 | 24.75 | 28.5 | 32.25 | 36 | 39.75 | 43.5 | 47.25 | 51 | 54.75 | 58.5 | 62.25 |
| 9 | 23 | 27 | 31 | 35 | 39 | 43 | 47 | 51 | 55 | 59 | 63 | 67 |
| 10 | 25 | 29.25 | 33.5 | 37.75 | 42 | 46.25 | 50.5 | 54.75 | 59 | 63.25 | 67.5 | 71.75 |

Data Set 2:

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.6796 | 1.0873 | 1.4951 | 1.9028 | 2.3105 | 2.7183 | 3.126 | 3.5338 | 3.9415 | 4.3493 | 4.757 | 5.1647 |
| 2 | 1.8473 | 2.9556 | 4.064 | 5.1723 | 6.2807 | 7.3891 | 8.4974 | 9.6058 | 10.714 | 11.822 | 12.931 | 14.039 |
| 3 | 5.0214 | 8.0342 | 11.047 | 14.06 | 17.073 | 20.086 | 23.098 | 26.111 | 29.124 | 32.137 | 35.15 | 38.163 |
| 4 | 13.65 | 21.839 | 30.029 | 38.219 | 46.408 | 54.598 | 62.788 | 70.978 | 79.167 | 87.357 | 95.547 | 103.74 |
| 5 | 37.103 | 59.365 | 81.627 | 103.89 | 126.15 | 148.41 | 170.68 | 192.94 | 215.2 | 237.46 | 259.72 | 281.99 |
| 6 | 100.86 | 161.37 | 221.89 | 282.4 | 342.91 | 403.43 | 463.94 | 524.46 | 584.97 | 645.49 | 706 | 766.51 |
| 7 | 274.16 | 438.65 | 603.15 | 767.64 | 932.14 | 1096.6 | 1261.1 | 1425.6 | 1590.1 | 1754.6 | 1919.1 | 2083.6 |
| 8 | 745.24 | 1192.4 | 1639.5 | 2086.7 | 2533.8 | 2981 | 3428.1 | 3875.2 | 4322.4 | 4769.5 | 5216.7 | 5663.8 |
| 9 | 2025.8 | 3241.2 | 4456.7 | 5672.2 | 6887.6 | 8103.1 | 9318.5 | 10534 | 11749 | 12965 | 14180 | 15396 |
| 10 | 5506.6 | 8810.6 | 12115 | 15419 | 18722 | 22026 | 25330 | 28634 | 31938 | 35242 | 38546 | 41850 |

Data Set 3:

| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.25 | 1.4 | 1.55 | 1.7 | 1.85 | 2 | 2.15 | 2.3 | 2.45 | 2.6 | 2.75 | 2.9 |
| 2 | 1.7678 | 2.1968 | 2.6987 | 3.2842 | 3.9656 | 4.7568 | 5.6739 | 6.7348 | 7.9601 | 9.373 | 11 | 12.871 |
| 3 | 2.1651 | 2.8593 | 3.7327 | 4.8274 | 6.1945 | 7.8964 | 10.009 | 12.626 | 15.859 | 19.845 | 24.75 | 30.776 |
| 4 | 2.5 | 3.4472 | 4.6987 | 6.3446 | 8.5004 | 11.314 | 14.973 | 19.721 | 25.862 | 33.79 | 44 | 57.125 |
| 5 | 2.7951 | 3.9853 | 5.617 | 7.8428 | 10.865 | 14.953 | 20.464 | 27.87 | 37.793 | 51.058 | 68.75 | 92.296 |
| 6 | 3.0619 | 4.4867 | 6.4991 | 9.3259 | 13.278 | 18.781 | 26.415 | 36.971 | 51.526 | 71.541 | 99 | 136.59 |
| 7 | 3.3072 | 4.9595 | 7.3521 | 10.797 | 15.732 | 22.772 | 32.777 | 46.949 | 66.963 | 95.149 | 134.75 | 190.26 |
| 8 | 3.5355 | 5.4092 | 8.1809 | 12.257 | 18.221 | 26.909 | 39.515 | 57.745 | 84.027 | 121.81 | 176 | 253.54 |
| 9 | 3.75 | 5.8396 | 8.9893 | 13.708 | 20.741 | 31.177 | 46.599 | 69.311 | 102.65 | 151.47 | 222.75 | 326.6 |
| 10 | 3.9528 | 6.2536 | 9.7798 | 15.151 | 23.29 | 35.566 | 54.006 | 81.607 | 122.79 | 184.07 | 275 | 409.64 |

**Data Set Assigned:** ____________________

| **Data Set** | **Type of Trend-line** | **Equation** | $R^2$ **Value** |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |

**For each of the following questions, graph the data and attach the graph to the back of the lab.**

**Problem 3 - Charles Law**

**Charles Law** states that the volume of a gas directly proportional to the temperature of the gas. Using the following temperature and volume data, determine the slope of the line for methane ($CH_4$) gas. Note, the data has negative values, you will need to adjust your axis so that we only see 1 quadrant.

| **Temperature (℃)** | Volume (mL) |
|---|---|
| -15 | 4.4 |
| 0.0 | 4.6 |
| 21 | 5.0 |
| 48 | 5.4 |
| 80. | 5.9 |

1. An interesting feature of a Charles Law plot is that extrapolation of the data to 0 volume for many different molecules all gives the same temperature, which is the lowest possible temperature possible (commonly referred to as Absolute Zero for the Kelvin temperature scale). Using your data, what is the value of Absolute Zero in Celsius? (Find the temperature when the volume is 0 mL)? How does your calculated value compare to the real value (you may need to google absolute zero)?

2. Calculate the volume (mL) of the Methane gas ($CH_4$) at 300.0 ℃.

**Problem 4 - Cell Phones**

The table below gives the number y (in millions) of cell-phone subscribers from 1988 to 1997 where $t$ is the number of years since 1987. Graph the data, and use an exponential trend-line to fit the data.

| time since 1987(year) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| # cell phone users (millions) | 1.6 | 2.7 | 4.4 | 6.4 | 8.9 | 13.1 | 19.3 | 28.2 | 38.2 | 48.7 |

1. Using the equation of the line how many millions of cell phones will there be in 15 years?

2. In what year will the number of cell phone users reach 2 billion (ie 2,000 million)?

**Graph 5 - Astronomy**

The table below gives the mean distance from the sun (in astronomical units [AU] and the period of each planet (time it takes to circle the sun) in years for the six planets closed to the sun. (One astronomical unit = 92 955 807.3 miles) Graph the data, and use an power trend-line to fit the data.

| Planet | Mercury | Venus | Earth | Mars | Jupiter | Saturn |
|---|---|---|---|---|---|---|
| Distance (AU) | 0.387 | 0.723 | 1.000 | 1.524 | 5.203 | 9.539 |
| Period (years) | 0.241 | 0.615 | 1.000 | 1.881 | 11.862 | 29.458 |

1. Neptune is 30.1 AU away from the sun, how many years does it take to circle the sun?

2. Careful measurements of the orbit of Pluto indicates that it take 248.6 years to circle the sun. How far from the sun (in AU) is Pluto? How far from the sun is Pluto in miles?